

DSN Telemetry System Performance With Convolutionally Coded Data

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DSN Telemetry System performance in decoding convolutionally coded data by both sequential and maximum likelihood techniques is being determined by testing at various Deep Space Stations. The evaluation of performance models is also an objective of this activity. This article reports the results obtained to date and the plans for future experiments.

I. Introduction

The Deep Space Network currently performs sequential decoding of Pioneer and Helios spacecraft convolutionally coded telemetry data using the Fano algorithm. The Network is implementing maximum likelihood decoding using the Viterbi algorithm for outer-planet Mariner spacecraft such as the Mariner Jupiter-Saturn. For the telecommunication system design the characteristics of these decoders must be known. This report describes the test activities presently in progress, the results obtained to date, and the plans for the near future to complete the specification of Network performance in sequential and maximum likelihood decoding.

II. Sequential Decoding

The objectives of these tests are to determine DSN Telemetry System performance. Specifically, frame deletion rate, probability of undetected bit errors, and optimum modulation index as a function of carrier power, data power, and data rate are the performance parameters being sought.

Sequential decoding performance has been investigated for some time since the effects of the noisy carrier reference on the decoder were first appreciated (Ref. 1). Tests were performed at various stations by both JPL and Ames Research Center personnel to develop empirical models

and to verify the theoretical models described in Refs. 2 and 3. These tests were the basis for the performance specification published in the current Deep Space Network/Flight Project Interface Design Handbook, 810-5, Revision D. However, the specification has not been completed for data rates below 512 bps.

Both Pioneer and Helios spacecraft use data rates down to 8 bps. The difficulty in obtaining results for low data rates arises when a long telemetry frame is used together with a small frame deletion rate (FDR). For example, past tests were run to obtain frame deletion rates of 10^{-4} with a frame length of 1152 bits, the Helios frame length.

The tests described in this report attempt to overcome this problem by performing two types of tests. The first type employs the sequential decoder implemented in the Data Decoder Assembly (DDA) (Fig. 1). These tests are called on-line decoding since the decoding is performed in real-time. Only the computations per frame are recorded. These data are subsequently processed and the plots shown in Figs. 2-9. The on-line tests are run with a relatively short frame length of 192 bits to obtain results rapidly.

The second type of tests are referred to as off-line tests. The results of the analog-to-digital conversion of each symbol are recorded. The data are later decoded in a sequential decoding program written for the Univac 1108.¹ The off-line decoding can be performed for various frame lengths and the effect of frame length on performance determined. By this means, the results obtained from the on-line test can be related to other frame lengths.

Four other advantages of the off-line tests are that undetected bit errors produced by the decoder can be monitored, the effect of the resolution of the A-D conversion on performance can be studied, the computational cutoff can be extended, and sequential decoding can be compared to maximum likelihood decoding.

The test plan employed a pattern which can be decoded by both the sequential decoder used for Pioneer and the maximum likelihood decoder planned for Mariner Jupiter-Saturn (MJS). Thus the identical off-line data can be decoded by both decoders and the results compared.

¹Based on a program supplied by L. Hofman of the NASA Ames Research Center.

III. Maximum Likelihood Decoding

The objectives of the maximum likelihood decoding tests are to establish the DSN Telemetry System performance in this mode of operation. For this purpose a Univac 1108 computer program² was developed (Ref. 4). This program uses the Viterbi algorithm to determine the characteristic of the decoder—specifically, bit error rate (BER), best branch metric decision vs majority vote, burst error length and density, length of error-free runs, decoder synchronization time, and optimum modulation index as a function of carrier and data power.

Tests currently under way in Spain at DSS 62 are being performed using a commercial Viterbi decoder (Ref. 5). In addition to the objectives listed, these tests will determine the relation between the performance monitor (normalization rate) provided by the decoder and signal-to-noise ratio (SNR).

IV. Test Procedures

One difficulty in performing tests in a Deep Space Station or in the Compatibility Test Area (CTA 21) is establishing the signal-to-noise ratio exactly. To avoid this uncertainty, errors in the coded symbols are monitored by both the Viterbi and the sequential decoding programs, and the symbol error rate is computed. The symbol error rate is used as the independent variable of the observation. This rate can be converted to a signal-to-noise ratio (energy per bit over noise spectral density) by using the complementary error function to obtain the ST_b/N_0 (E_b/N_0) presented to the decoder. Modeling of the receiver, demodulator, and symbol synchronizer losses is required to calculate the ST_b/N_0 presented to the front end of the DSN telemetry system.

Figure 1 shows the test setup for both the on-line and off-line tests. The simulation Conversion Assembly (SCA) generates a symbol stream which simulates both a Pioneer 32:1/2 and the MJS 7:1/2 codes. The data stream is modulated on the subcarrier and passed to the test transmitter, where the carrier is modulated. A Y-factor is used to measure signal-to-noise ratio (Ref. 6). The data passes through the microwave equipment (UWV) to the receiver and demodulator. At the output of the Subcarrier Demodulator Assembly (SDA) the stream is split into two branches. In one branch, the on-line test, the symbols

²Based on a program supplied by B. Baston of the NASA Johnson Space Center.

are synchronized, the data are decoded sequentially, and the computations per frame are recorded.

The second branch, the off-line test, is processed normally only up through the Symbol Synchronizer Assembly (SSA). The eight-bit A-D conversion of each symbol is recorded on magnetic tape for subsequent decoding and analysis.

Table 1 lists the test conditions used for medium-to-low-rate sequential and maximum likelihood decoding tests. Modulation indices in these tests were selected near theoretical optimum to determine the actual optimum. The test conditions A through F listed in Table 1 are primarily oriented to Pioneer and Helios. Later tests are MJS-oriented and are also intended to determine the effect of Block III vs Block IV receiver performance on both types of decoding.

V. Test Anomalies

Significant difficulty was experienced in reading magnetic tapes written by the XDS-920 into the Univac 1108. A special tape reading program³ was used to overcome numerous parity errors on the test data tapes. Checks were performed by preprocessing programs to eliminate bad records so that results would not be contaminated.

The importance of balancing the SDA-SSA interface became apparent when the distribution of symbol errors was monitored. In cases where SDA balance was off, the frequency of ones in error far outweighed the zeros in error. Consequently, SDA balance calibrations were performed before each test.

One anomaly, which has not been explained, was the occurrence of a more frequent symbol error in one symbol position. A six-symbol pattern was generated by the six binary-coded fixed toggle switches on the Simulation Conversion Assembly front panel. In one test, the first symbol accounted for 80% of the errors in the entire file. This anomaly has never reappeared. The test data were not used.

VI. Sequential Decoding

This section presents an account of some of the sequential decoding tests.

³Written by R. B. Hartley, TDA Planning Section.

A. Decoding Program

The U1108 sequential decoding program is a straightforward implementation of the basic Fano algorithm described in Ref. 7. Following are some properties of the program:

- (1) *Computation count* increases by 1 each time the decoder looks forward one step or backward one step in the code tree.
- (2) *Frame length* must be a multiple of 36. Since, for these tests, the on-line decoder frame length was set to 192 bits (384 symbols), we used a frame length of 180 for most runs. One file was run with a range of frame lengths from 180 to 1152 to test dependence of results of this parameter.
- (3) The *tail sequence* is just the last 32 bits of each frame. Thus, for a 180-bit frame, only the first 148 decoded bits are checked against the known pattern for "undetected" bit errors.
- (4) If the computation count for a frame exceeds the preset *computational cutoff*, that frame is declared erased. For on-line decoding, the cutoff is inversely proportional to data rate. For off-line decoding, it was often not economical to use the on-line value of cutoff, because erased frames cost much central processing unit (CPU) time. For most tests, a default value of 100 computations per bit was used.
- (5) A running tabulation of hard-decision *symbol error rate* (SER) is printed by the program.

B. Format of Performance Measures

The tests are identified by data rate in bits per second (bps), modulation index (MI), and signal-to-noise ratio (SNR) $E_b/N_0 = ST_b/N_0$ as seen by the decoder. SNR at the input of the receiver is difficult to determine accurately, but hard decision SER was directly observed. Thus we chose to estimate decoder E_b/N_0 from observed probability P_E of symbol errors by inverting the usual complementary error function formula

$$P_E = \operatorname{erfc}(\sqrt{2E_s/N_0}) = \operatorname{erfc}(\sqrt{E_b/N_0})$$

where

$$\operatorname{erfc}(x) = (2\pi)^{-1/2} \int_x^\infty \exp\left(-\frac{1}{2}t^2\right) dt$$

Graphical display of decoding performance is achieved by a log-log plot of the sample distribution function of the number of computations per bit for all frames decoded

or erased. The horizontal coordinate is the number of computations L per bit (number of computations divided by frame length), and the vertical coordinate is the fraction of frames such that the number of computations equals or exceeds L .

We have also plotted some linear-scale histograms of the above sample distribution, normalized so that the total area under the histogram is 1. They give a closer look at the distributions near the point of maximum density.

Other performance parameters shown are mean number of computations per bit (CPB) and undetected bit error rate (BER). Some of the sample distributions have rather long, heavy tails. This causes mean CPB to depend strongly on computational cutoff. Therefore, in calculating mean CPB, each distribution was truncated at 100 CPB; that is, any datum greater than 100 was set equal to 100.

C. Comparison of 64-bps Tests

Figure 2 and Table 2 compare six tests at this data rate. Mean CPB and the distribution function values increase as E_b/N_0 decreases, except for test D6. This test has a relatively high modulation index, and the Telemetry Analysis Program of G. Dunn shows a receiver SNR degradation of 2.4 dB. The symbol errors, although scanty, appear to occur in bursts; this forces the decoder to make long searches.

The "undetected" bit errors invariably occur in bursts near the end of the 148-bit data portion of the 180-bit frame. Apparently, if a burst of bad symbols occurs at about this point, it is sometimes possible for the decoder to reach the end of the frame before these errors have been corrected or the computational cutoff has been reached.

D. Comparison of CTA 21 and DSS 62 Results

As previously described, on-line and off-line tests were performed at CTA 21. To investigate station-to-station variation in performance, on-line tests at 8 bps were performed at Madrid, DSS 62. The Madrid station used a pseudorandom data source, encoded with the 32:1/2 code. On the other hand, the CTA 21 tests used a short repetitive pattern since these tests had the dual purposes of sequential and maximum likelihood decoding performance.

The DSS 62 data were reduced, analyzed, and compiled on site by station personnel.

Figure 3 and Table 3 contain results from the two stations. At CTA 21, only tests F1, F2, and F3 have been reduced. The distribution functions from DSS 62 are enclosed by 90% confidence intervals for the whole 32-h test (lightweight solid curves), and by 1σ intervals for a 2-h subtest (dashed curves). These are shown here for tests F4 and F5. We have omitted them from the F1, F2, and F3 plots to avoid overcrowding these figures, which also contain the CTA 21 curves.

The main difference between the DSS 62 and CTA 21 results is the shape of the distributions near $L = 1$. Each CTA 21 distribution function has a knee there, but the DSS 62 functions have none. This means that the CTA 21 tests yielded fewer "very good" frames. The histograms of the CTA 21 tests (Fig. 4) show that the most likely number of computations per bit is about 1.6 for F1-3, 1.3 for F2, and 1.1 for F3. At DSS 62 the most likely value appears to be 1 for these tests.

E. Comparison with Lesh's Model

Figure 5 compares two of our off-line 64-bps tests with J. R. Lesh's theoretical model, which Lesh has implemented on the 1108 (Ref. 3). The slope of the model agrees well with the slope of the experimental curve for more than 10 computations per bit. The frame length is 180 bits; Ref. 3 reported that the model fits long-frame-length data better than short. In the future, we will compare Lesh's model to off-line decoding at frame lengths up to 1152.

F. Different Frame Lengths

We decoded the same symbol stream with frame lengths of 180, 216, 504, and 1152. Figure 6 shows the distribution functions, and Figure 7 shows the histograms. The data rate is 128 bps and SER is 2.32%, from which decoder E_b/N_0 is deduced to be 5.98 dB. There were no bit errors. Mean computations per bit are 1.146, 1.145, 1.148, 1.148, respectively.

The distributions keep the same mean, but as the frame length increases, they tend to become more symmetric. This behavior resembles that of a model in which the number of computations for a frame is the sum of a collection of identically distributed, independent, random variables with finite variance. The longer the frame, the more of these variables are summed, and the more the distribution of the sum approaches a Gaussian. It would be desirable to repeat this experiment with (1) a longer symbol stream, to get better statistics on the longer frame

lengths, and (2) a test in which the distributions have such long, heavy tails that, when extrapolated indefinitely far, they have infinite variance or mean.

Looking at the results for 180-bit and 216-bit frames, we conclude that either is an acceptable substitute for the 192-bit frame.

VII. Maximum Likelihood Convolutional Decoding

In addition to the sequential decoding tests, the off-line test data have been used to investigate maximum likelihood decoding. These tests collect Symbol Synchronizer Assembly (SSA) output data as shown in Fig. 1. The data are read into the Univac 1108, where a program (Ref. 4) decodes the data, measures symbol and bit error rate, error burst statistics, and the distribution of error-free runs. A user's guide to the program is given in Ref. 8.

To verify the correctness of the implementation of the Viterbi algorithm in the Univac 1108 program as a simulation of the maximum likelihood convolutional decoder (MCD) being implemented in the Network, a series of simulation cases were run. A Univac 1108 program (CODER, Ref. 8) was used to produce simulated output data corrupted by additive white Gaussian noise (AWGN). These data were decoded and the results are compared (see Table 4 and Fig. 8) to the prototype MCD acceptance test results. Since the prototype decoder acceptance tests were performed presumably with AWGN, this type of noise was applied to the simulated data to establish the correctness of the simulated decoder.

The Univac 1108 Viterbi decoding program has the capability of using two different criteria to choose the most likely branch and, hence, decode each bit correctly. First, all surviving branches can be polled and a majority vote taken. Second, the surviving branch with the best state metric can be selected. Figure 8 and Table 4 compare these criteria when AWGN is applied.

It is interesting to note that when the best metric criteria are used, ties frequently occur even with 3-bit quantized symbols. An early implementation of the Univac 1108 Viterbi decoding program would select the first of the tying branches encountered. This resulted in the best metric criterion performing more poorly than the majority vote criterion. When the program was modified to select the last of the tying branches, the best

metric criterion performed better than the majority vote criterion and compared very well to the prototype decoder acceptance test results.

It is believed that this sensitivity of the best metric criterion to resolving ties is due to the data pattern. A nonrandom data pattern was used which causes the correct branches to be a certain few in one part of the trellis. If the trellis is always searched from the top down and if the first tying best-state metric is always selected, then the performance of the decoder can be biased, depending on whether the true branches are near the top or the bottom of the trellis. Consequently, ties in best-best metrics should be resolved randomly if the data are not random.

Figure 9 provides a comparison of an 1108 simulation curve using the receiver quantization threshold spacing $0.5\sqrt{N_0/2}$, the functional requirements, and the prototype decoder acceptance test results. The threshold spacing of the data obtained from CTA 21 has been calculated to be in the neighborhood of $0.5\sqrt{N_0/2}$.

To investigate decoder performance in the DSN Telemetry System and, in particular, the effect of correlated noise produced by the receiver phase error, test data from CTA 21 were decoded and the results to date listed in Table 5. These tests show that DSN telemetry system functional requirements are being met. As in the case of sequential decoding performed on the U1108, the symbol error rate was monitored and used as the independent variable of the observation. Symbol error rate was converted to E_b/N_0 at the input to the decoder by the complementary error function.

VIII. Future Activities

Major future testing activities include:

- (1) Determining the effect on performance of sequential and maximum likelihood decoders of Block III receivers and demodulators vs Block IV.
- (2) Determining performance of the decoders for S- vs X-band.
- (3) Determining MJS '77 encounter data rates using X-band performance with maximum likelihood decoding.
- (4) Completion of MJS '77 cruise data rates currently under way at DSS 62.

References

1. Layland, J. W., "A Sequential Decoding Medium Rate Performance Model," in *The Deep Space Network*, Technical Report 32-1526, Vol. XVIII, pp. 29-40, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1973.
2. Layland, J. W., "DSS Tests of Sequential Decoding Performance," in *The Deep Space Network Progress Report 42-20*, pp. 69-77, Jet Propulsion Laboratory, Pasadena, Calif., Apr. 15, 1974.
3. Lesh, J. R., "Sequential Decoding in the Presence of a Noisy Carrier Reference," in *The Deep Space Network Progress Report 42-23*, pp. 111-124, Jet Propulsion Laboratory, Pasadena, Calif., Oct. 15, 1974.
4. Benjauthrit, B., Mulhall, B. D. L., and Wong, J. S., "A Viterbi Decoding Program for DSN Telemetry System Analysis," in *The Deep Space Network Progress Report 42-28*, pp. 5-10, Jet Propulsion Laboratory, Pasadena, Calif., Aug. 15, 1975.
5. *The LV7015 Reference Manual*, LINKABIT Corp., San Diego, Calif. 1972.
6. Baumgartner, W. S., et al., "Multiple-Mission Telemetry System Project," in *The Deep Space Network*, Space Program Summary 37-60, Vol. II, pp. 152-169, Jet Propulsion Laboratory, Pasadena, Calif., Nov. 30, 1969.
7. Wozencraft, Z. M., and Jacobs, I. M., *Principles of Communication Engineering*, John Wiley and Sons, Inc., New York, 1965.
8. Benjauthrit, B., Mulhall, B. D. L., and Wong, J. S., "A Viterbi Decoding Program for Telemetry System Analysis (User's Guide), "IOM AE-75-21, Jet Propulsion Laboratory, Pasadena, Calif., Sept. 3, 1975 (an internal document).

Table 1. Convolutional decoding test plan

Test	Data rate (SPS)	Modu- lation index, (deg)	P_T/N_0 , dB sec ⁻¹	E_s/N_0 , dB	Degraded E_s/N_0 , dB	SSA estimate, dB	SER, %
A1	4096	55.0	39.0	1.14	0.95	1.96	5.74
A2	4096	55.0	39.5	1.64	1.46	2.28	4.71
A3	4096	55.0	40.0	2.14	1.98	2.62	3.79
A4	4096	55.0	40.5	2.64	2.49	2.98	2.98
A6	4096	67.6	39.5	2.69	2.42	2.94	3.07
A7	4096	75.0	39.5	3.08	2.64	3.09	2.77
B1	2048	55.0	36.0	1.15	0.85	1.91	5.95
B2	2048	55.0	36.5	1.65	1.37	2.22	4.88
B3	2048	55.0	37.0	2.15	1.89	2.57	3.93
B4	2048	55.0	37.5	2.65	2.41	2.93	3.09
B6	2048	67.6	36.5	2.71	2.27	2.83	3.31
B7	2048	75.0	36.5	3.09	2.34	2.87	3.21
C1	256	55.0	28.4	2.56	1.59	2.37	4.46
C2	256	55.0	29.2	3.38	2.52	3.00	2.94
C3	256	55.0	30.0	4.18	3.40	3.67	1.82
C4	256	42.0	29.2	1.63	1.00	1.99	5.63
C5	256	67.6	29.2	4.44	2.67	3.12	2.72
D1	128	55.0	26.4	3.60	2.21	2.78	3.41
D2	128	55.0	27.2	4.40	3.14	3.47	2.11
D3	128	55.0	28.0	5.20	4.06	4.19	1.20
D4	128	42.0	27.2	2.64	1.80	2.50	4.10
D5	128	58.4	27.2	4.73	3.24	3.54	2.00
D6	128	66.5	27.2	5.38	2.79	3.20	2.56
E1	32	42.0	21.4	2.86	0.72	1.86	6.22
E2	32	42.0	22.2	3.66	1.78	2.52	4.12
E3	32	42.0	23.0	4.46	2.80	3.23	2.54
E4	32	37.2	22.7	3.31	1.77	2.51	4.15
E5	32	45.0	22.2	4.14	2.09	2.72	3.61
F1	16	42.0	19.4	3.87	0.78	1.89	6.08
F2	16	42.0	20.0	4.47	1.72	2.48	4.23
F3	16	42.0	20.6	5.07	2.61	3.09	2.81
F4	16	37.2	20.5	4.12	1.92	2.61	3.88
F5	16	45.0	20.0	4.95	1.92	2.61	3.88

Table 2. Off-line sequential decoding of 64-bps tests

Test identi- fication	Modu- lation index, deg	SER, %	E_b/N_0^a , dB	Mean CPB	BER
D1	55.0	3.70	5.04	1.602	0
D2	55.0	2.98	5.50	1.437	6×10^{-6}
D3	55.0	1.61	6.61	1.078	0
D4	42.0	6.62	3.55	8.98	1.6×10^{-4}
D5	58.4	2.06	6.20	1.240	6×10^{-6}
D6	66.5	2.52	5.83	5.26	1.3×10^{-4}

^aDecoder SNR deduced from observed SER.

Table 3. Off-line sequential decoding of 8-bps tests at CTA 21 and DSS 62

Test identi- fication (station)	Modu- lation index, deg	SER, %	E_b/N_0^a , dB	Mean CPB	BER
F1-1 (21)	42.0	7.28	3.26	10.4	1.6×10^{-3}
F1-3 (21)	42.0	6.32	3.68	2.96	2.5×10^{-5}
F1 (62)	42.0	6.43	3.64		0 ^b
F2 (21)	42.0	4.98	4.34	1.888	0
F2 (62)	42.0	4.14	4.79		0
F3 (21)	42.0	3.91	4.92	1.327	0
F3 (62)	42.0	2.69	5.71		0
F4 (62)	37.2	3.73	5.02		0
F5 (62)	45.0	3.10	5.42		0

^aDecoder SNR deduced from observed SER.

^bOne blunder run excluded.

Table 5. Preliminary results

Test	E_b/N_0 , dB	Errors	Errors, maj	Error rate			Total			Average, bits		
				BER, $\times 10^{-4}$	BER, maj, $\times 10^{-4}$	SER, %	Bits	Bursts	Gaps	Errors/ burst	Burst length	Gap length
E1	3.28	16	78	5.079	8.938	6.8	155520	16	78	4.7	8.5	1688.6
F1	3.69	19	41	1.290	2.790	6.3	147132	5	20	3.4	5.4	7355.7
F2	4.34	22	22	1.370	1.370	5.0	160236	5	23	4.4	6.2	6965.8
A2	4.57	3	7	0.043	0.100	4.5	699840	1	4	3.0	3.0	99918.5
E5	4.73	10	12	0.429	0.514	4.5	233280	2	11	3.0	4.0	21206.4
A3	4.89	5	5	0.058	0.058	4.2	855360	1	6	5.0	7.0	66538.0

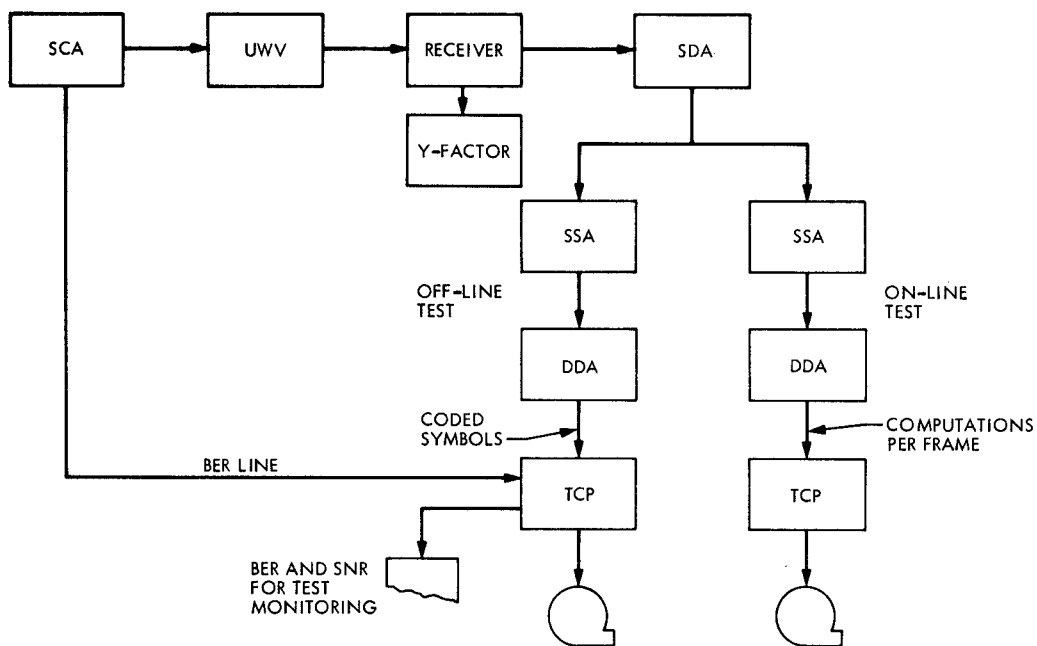


Fig. 1. Test configuration for DSN Telemetry System performance with convolutional codes

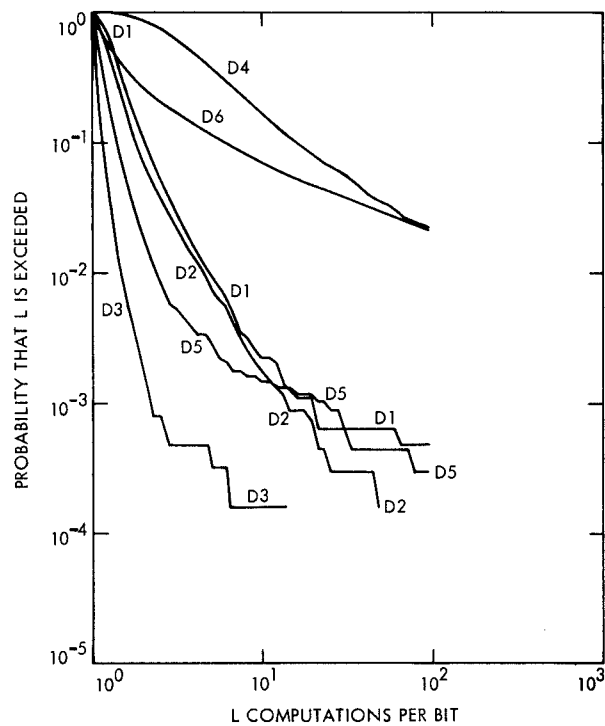


Fig. 2. Off-line sequential coding of 64-bps tests

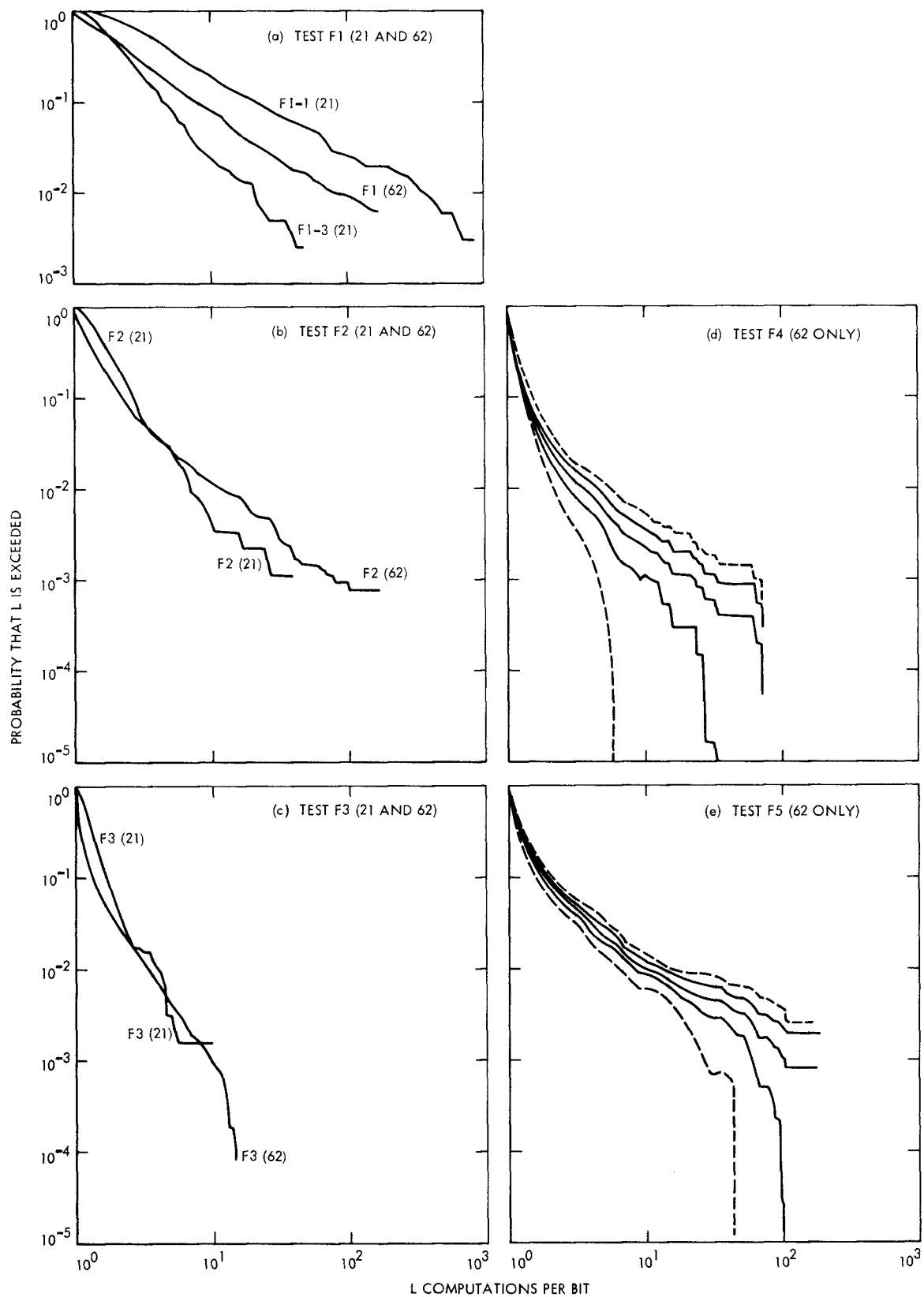


Fig. 3. The 8-bps tests at CTA 21 and DSS 62

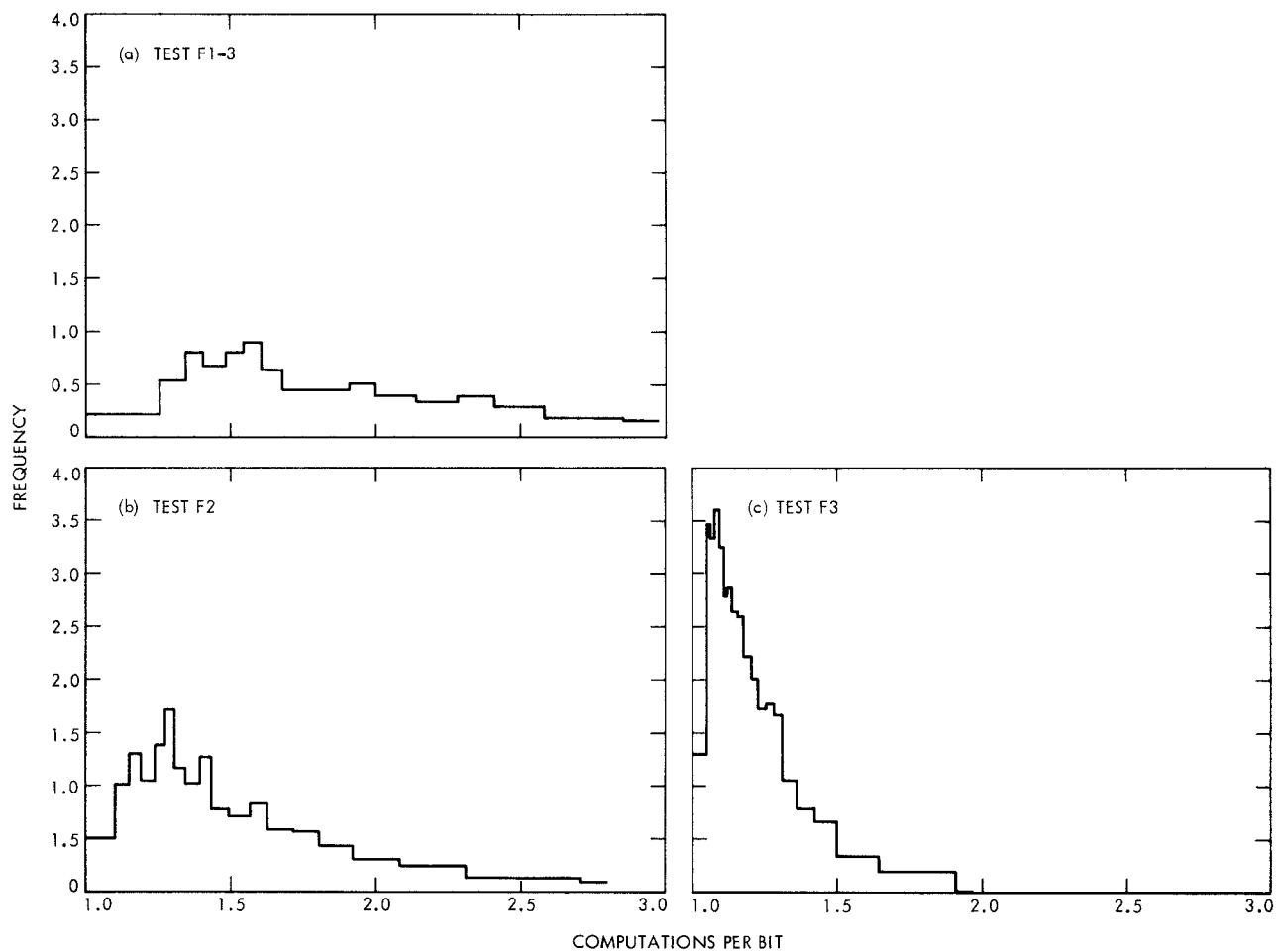


Fig. 4. Histograms of sequential decoding of 8-bps tests from CTA 21

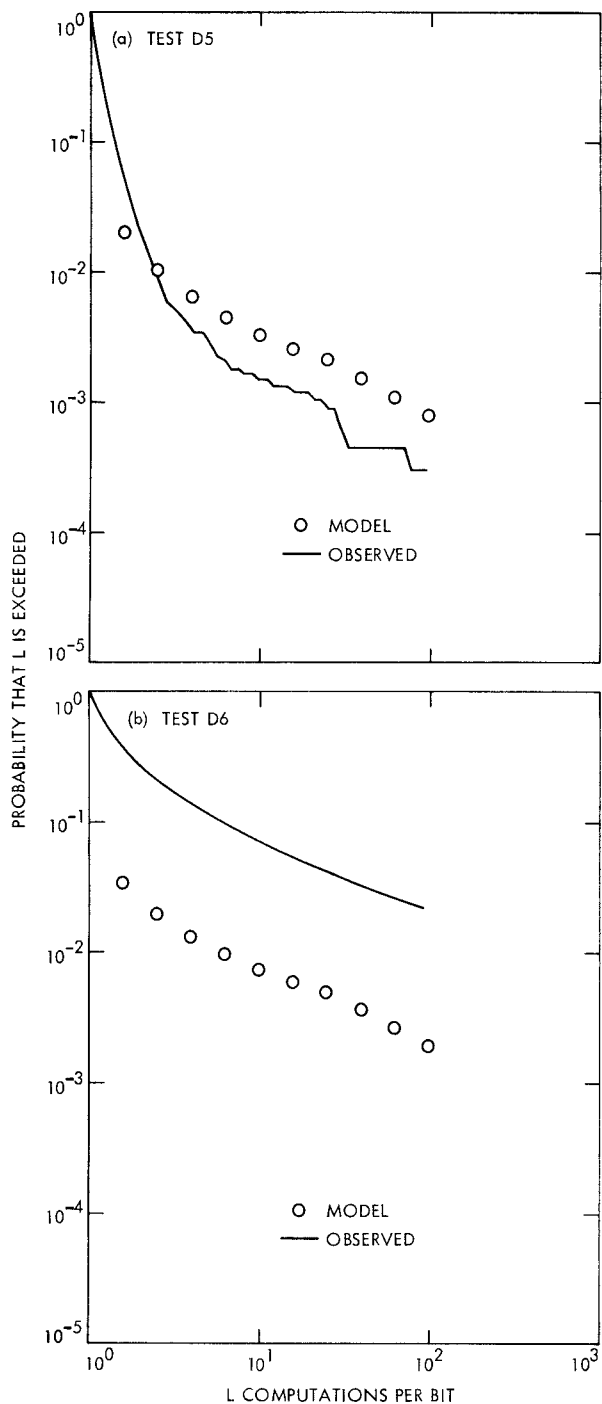


Fig. 5. Comparison of 64-bps tests with Lesh's model

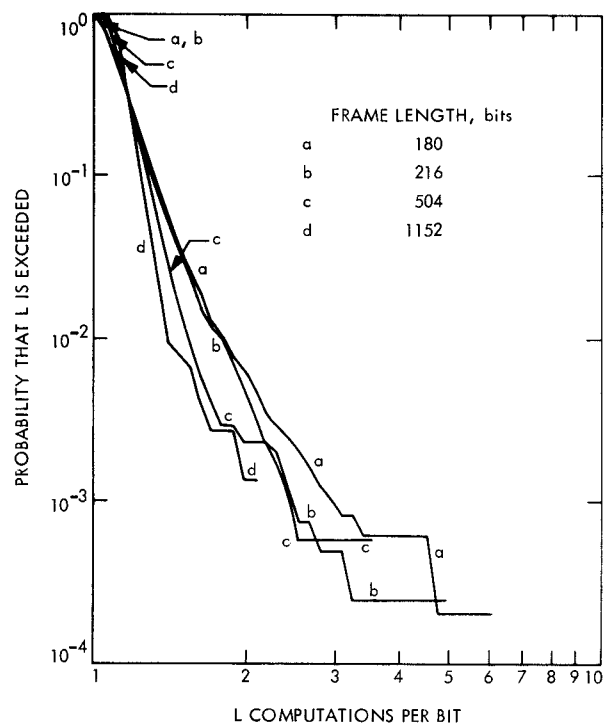


Fig. 6. Sequential decoding with different frame lengths: distribution functions

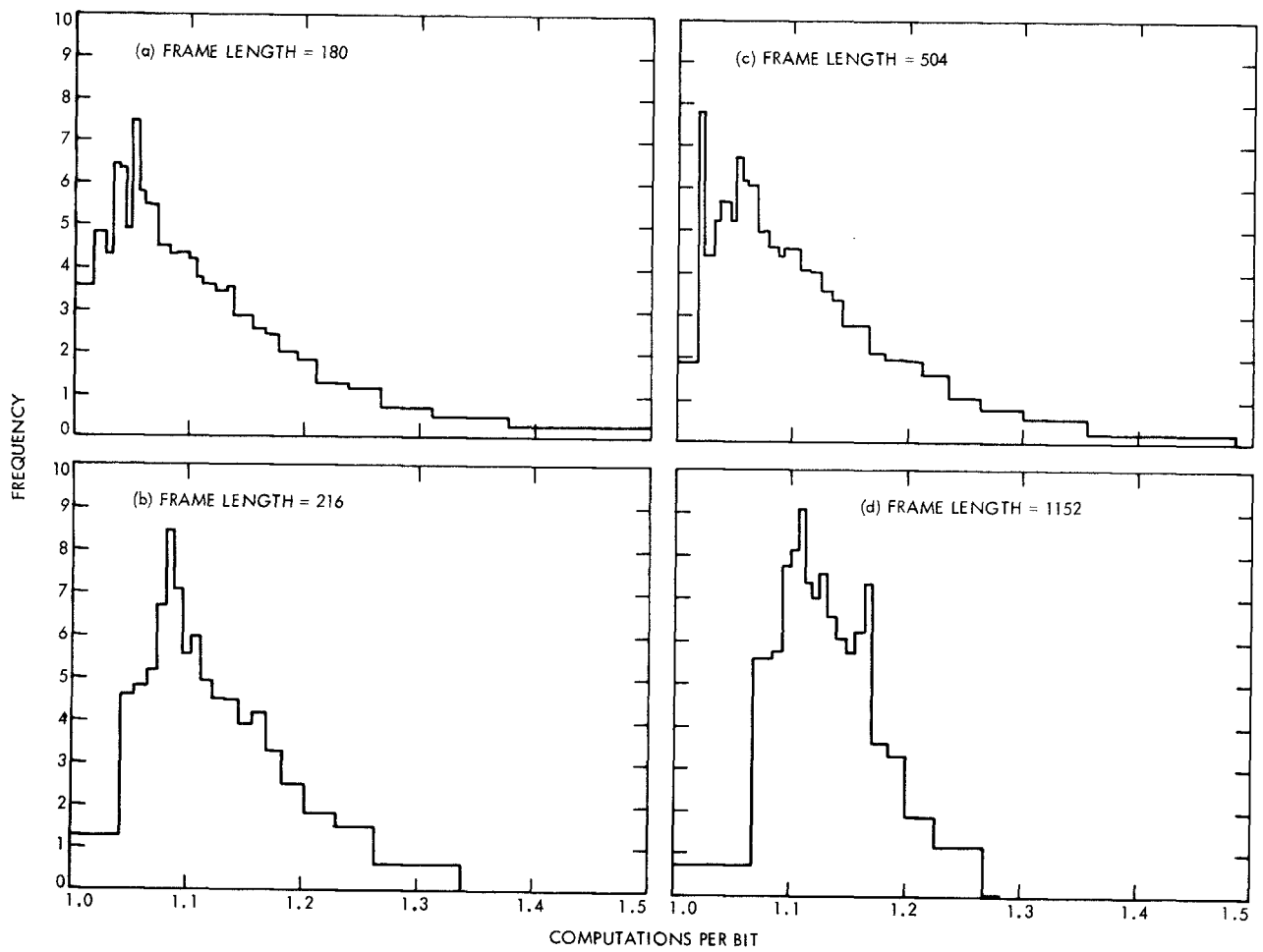


Fig. 7. Sequential decoding with different frame lengths: histograms

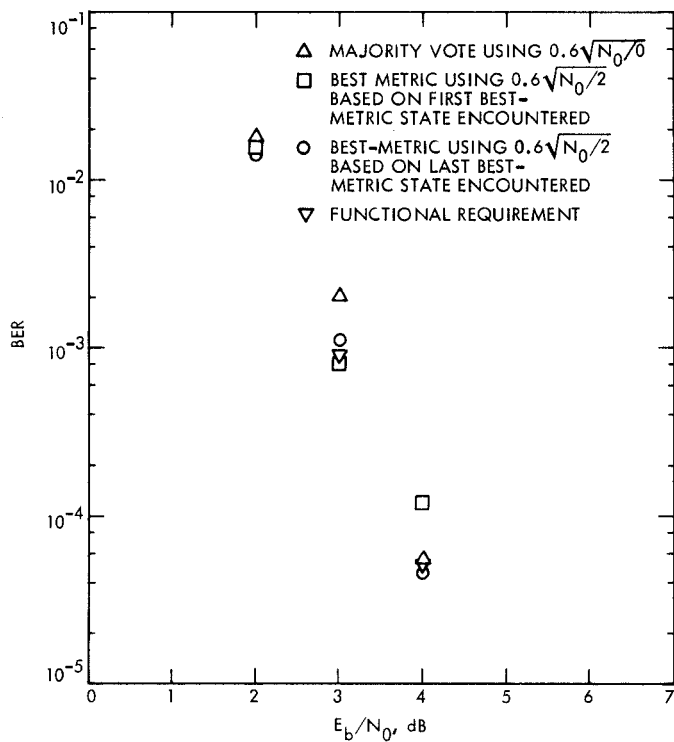


Fig. 8. Viterbi decoding program performance

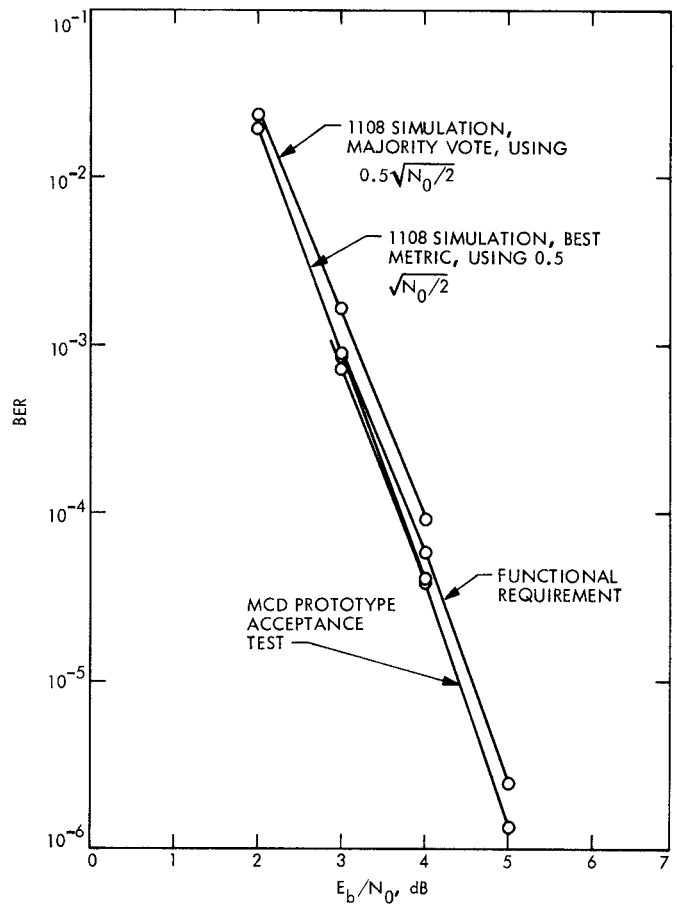


Fig. 9. Comparison of program performance, functional requirement E_b/N_0 (dB), and MCD prototype acceptance results